AGENT FORUM 2024: BOOK OF ABSTRACTS

The abstracts below are listed in alphabetical order.

ROOPE ANTTILA (UNIVERSITY OF OULU): ASSOUAD DIMENSIONS OF SELF-AFFINE SETS

One of the broad objectives in fractal geometry is classifying fractals, sets with complex and detailed structure at all scales, via different notions of dimension. Iterated function systems provide a simple and systematic way of constructing fractals while capturing much of their complicated behavior. In this talk I will discuss the Assouad dimension, which quantifies the size of the thickest parts of a set, in the context of affine iterated function systems on the plane. My aim is to motivate the study of Assouad dimension in this context and provide an intuitive explanation for a connection between the Assouad dimension of self-affine sets and the dimensions of their projections and slices. The talk is partly based on a joint work with Balázs Bárány and Antti Käenmäki.

VEDANSH ARYA (UNIVERSITY OF JYVÄSKYLÄ): OPTIMAL REGULARITY FOR THE VARIABLE COEFFICIENTS PARABOLIC SIGNORINI PROBLEM

In this talk, we discuss the optimal regularity of the variable coefficient parabolic Signorini problem with $W_p^{1,1}$ coefficients and L^p inhomogeneity, where $p > n + 2$ with n being the space dimension. Relying on a parabolic Carleman estimate and an epiperimetric inequality, we demonstrate the optimal regularity of the solutions as well as the regularity of the regular free boundary. This is based on joint work with Wenhui Shi.

ANTONIO COCAN (UNIVERSITY OF JYVÄSKYLÄ): HIGHER DIMENSIONAL SLOWNESS SURFACE DISTORTION

In this talk we study a side of an inverse problem in elastic geometry. A material can be characterized by its stiffness tensor and the time evolution of the displacement is modeled by the system of PDEs given by the matrix valued elastic wave equation. Propagating singularities are studied via microlocal analysis and its wavefront set. This leads to a polynomial equation in multiple variables and its roots characterize the propagating singularities. We call this associated zero set a slowness surface and it is an affine scheme - an object belonging to algebraic geometry. It was previously shown that if only conformal distortions are present, then we have uniqueness of stiffness tensors. In this talk we show that only conformal distortions of a slowness surface preserve it as a valid slowness surface. Our result holds in any dimension, but the existence of such a point is shown in dimensions two and three. The implications of these results is that we have no coordinate gauge freedom in elastic Finsler geometry in dimensions two and three.

ANNA DOLEŽALOVÁ (UNIVERSITY OF JYVÄSKYLÄ): APPROXIMATING WEAK LIMITS OF SOBOLEV HOMEOMORPHISMS

Can weak limits of homeomorphisms in Sobolev spaces be obtained as strong limits of homeomorphisms or even diffeomorphisms? This question has been answered in the planar case by [IO] and [DP]. Later, in [DHMa] we showed that this results do not generalize to the three-dimensional case at least for the space $W^{1,2}$. However, plenty of open questions remain. In this talk, we will have a close look at a counterexample leading to one of the negative results, and discuss an ongoing work with I. Kangasniemi and J. Onninen to generalize it.

MOHAMED FKIRINE (TAMPERE UNIVERSITY): GENERATOR PERTURBATIONS FOR STOCHASTIC CAUCHY PROBLEMS

In this talk, we delve into evolution equations perturbed by both noise and an unbounded perturbation. By rewriting these equations as stochastic Cauchy problems, we establish the existence and uniqueness of solutions. Furthermore, we explore the long-time behavior of the solutions, such as the absolute continuity and the existence of an invariant measure. This is a joint work with S. Hadd and A. Rhandi.

MIGUEL GARCÍA BRAVO (UNIVERSITY COMPLUTENSE OF MADRID): ABOUT THE GEOMETRY OF SOBOLEV EXTENSION DOMAINS

Given a domain $\Omega \subset \mathbb{R}^n$ and a Sobolev function of class $W^{1,p}$ it is of interest to study when such function admits an extension to the whole space $\mathbb{R}^n$ belonging to the same Sobolev class. In general the extension is possible whenever the domain has nice geometric properties, like having a Lipschitz boundary or being uniform (these results are due to Calderón, Stein, and Jones).

The main goal of this talk is to introduce the theory of Sobolev extension domains and briefly explain some necessary and sufficient geometric conditions satisfied by these domains that are known up to date. Properties like quasiconvexity or the Ahlfors n-regularity play a role at this point. We also want to put our attention on the boundary $\partial\Omega$ of Sobolev extension domains and study how big this set can be. In this direction we want to present two recent results:

- We want to show that, while for Sobolev extension domains $\Omega \subset \mathbb{R}^2$ that are homeomorphic to the unit disc we must have $\dim_{\mathcal{H}}(\partial\Omega) < 2$, there exists a domain $\Omega \subset \mathbb{R}^2$ homeomorphic to the unit ball that is a $W^{1,p}$-extension domain for every $1 \leqslant p \leqslant \infty$ but $\dim_{\mathcal{H}}(\partial\Omega) = 3$.
- We also inspect the possibility that the boundary of these domains may self-intersect. We conclude the talk by giving Hausdorff dimensional estimates on this so-called set of *two-sided points*

This is a joint work with Tapio Rajala and Jyrki Takanen.

MAX GOERING (UNIVERSITY OF JYVÄSKYLÄ): PRINCIPAL VALUES AND RECTIFIABILITY OF MEASURES

Since the work of Mattila and Preiss in 1995, it's been known that for a Radon measure with reasonable density assumptions, the almost everywhere existence of principal values of the Riesz transform is equivalent to the measure being rectifiable. This can be thought of as a positive solution to the qualitative analog of the David-Semmes conjecture. Recently, Molero, Mourgoglou, Puliatti, and Tolsa provided a new proof to the David-Semmes conjecture in codimension 1. In this talk, we discuss joint work with Casey, Toro, and Wilson on Λ-tangents, the role Λ-tangents serve in studying the geometry of measures through the lense of anisotropic principal values, and their relationship to the previously mentioned works.

JUHO HALONEN (UNIVERSITY OF EASTERN FINLAND): TROPICAL NEVANLINNA THEORY FOR TROPICAL MEROMORPHIC FUNCTIONS OF TWO VARIABLES

Classical Nevanlinna theory is used to measure complexity of meromorphic functions. Tropical Nevanlinna theory studies complexity of tropical meromorphic functions in an analogous way to classical Nevanlinna theory. In this talk I will introduce tropical Nevanlinna theory and present a way to extend it for tropical meromorphic functions of two variables.

SUSANNA HEIKKILÄ (UNIVERSITY OF HELSINKI): QUASIREGULAR CURVES AND COHOMOLOGY

In this talk, we define quasiregular ellipticity in the setting of quasiregular curves. We also discuss the cohomology of quasiregularly elliptic manifolds and give examples of (non-)ellipticity.

TONI IKONEN (UNIVERSITY OF HELSINKI): LIOUVILLE'S THEOREM FOR QUASIREGULAR CURVES

In dimension three or higher, every 1-quasiregular mapping between Euclidean domains is the restriction of a Möbius transformation. We discuss analogous results for quasiregular curves. More precisely, we ask whether or not the image of every 1-quasiregular curve is affine. The problem is connected to classification of conformally flat calibrated submanifolds of Euclidean spaces, and as such, the form used in the target plays a crucial role.

MIKKO JASKARI (UNIVERSITY OF TURKU): THE ZEROS OF THE RIEMANN ZETA FUNCTION AND PRIME NUMBERS

The Riemann zeta function is a very important function in number theory especially due to its connection prime numbers. If we understand the zeros of the zeta function we will also understand the distribution of prime numbers. In this talk we will discuss about the connection between the zeros and prime numbers and how research on the zeros could help us to obtain better bounds for prime number theorem.

SEONGMIN JEON (UNIVERSITY OF JYVÄSKYLÄ): CONVEXITY FOR A PARABOLIC FULLY NONLINEAR FREE BOUNDARY PROBLEM WITH SINGULAR TERM

In this talk, we study a parabolic free boundary problem in an exterior domain

$$\begin{cases} F(D^2u) - \partial_t u = u^a \chi_{\{u>0\}}, & \text{in } (\mathbb{R}^n \setminus K) \times (0,\infty), \\ u = u_0, & \text{on } \{t=0\}, \\ |\nabla u| = u = 0, & \text{on } \partial\Omega \cap (\mathbb{R}^n \times (0,\infty)), \\ u = 1, & \text{in } K \times [0,\infty). \end{cases}$$

Here, a belongs to the interval to the interval $(-1,0)$, K is a (given) convex compact set in $\mathbb{R}^n$, $\Omega = \{u > 0\} \supset K \times (0,\infty)$ is an unknown set, and F denotes a fully nonlinear opearator. Assuming a suitable condition on the initial value u_0, we prove the existence of a nonnegative quasiconcave solution to the aforementioned problem, which exhibits monotone non-decreasing behaviour over time. his is based on a joint work with Henrik Shahgholian.

RAHIM KARGAR (UNIVERSITY OF TURKU): FORMULAS FOR THE VISUAL ANGLE METRIC

During the past few decades, various intrinsic metrics on planar domains have become important tools in geometric function theory, for instance in the study of quasiconformal mappings. These metrics, defined in a general domain, on the one hand, share some of the properties of the hyperbolic metric on the unit disc and on the other hand, they are simpler than the hyperbolic metric. We study here one such intrinsic metric, the visual angle metric, introduced by Klén, Lindén, Vuorinen, and Wang in [KLVW].

Let G be a proper subdomain of $\mathbb{R}^n$ such that ∂G is not a proper subset of a line. The *visual angle metric* for $a, b \in G$ is defined by

$$v_G(a,b) = \sup\{\alpha : \alpha = \angle(a,z,b),\ z \in \partial G\}.$$

In this talk, we present several new formulas for the visual angle metric of the unit disc in terms of the hyperbolic metric. We also give a sharp Schwarz lemma for the visual angle metric under quasiregular mappings.

This talk is based on a joint paper with Fujimura and Vuorinen [FKV].

PETRI LAARNE (UNIVERSITY OF HELSINKI): PARADIFFERENTIAL CALCULUS AND DISPERSIVE PDES

In this talk, I will outline how Fourier-analytic methods can be used to solve certain nonlinear wave equations. In particular, I discuss how paradifferential calculus is used to make sense of rough, distribution-valued solutions relating to stochastic PDEs. This connects to e.g. Jean Bourgains work on harmonic analysis and PDE.

AAPO LAUKKARINEN (AALTO UNIVERSITY): CONVEX BODY DOMINATION OF CERTAIN MULTI-SCALE OPERATORS

The notion of sparse domination, i.e., dominating operators with averages taken over a sparse collection of cubes, has seen rapid development during the last decade. Convex body domination is a useful extension of this popular technique that lets one estimate operators in matrix-weighted spaces. In this talk, I will give all the relevant definitions and introduce a convex body domination principle for a certain class of multi-scale operators

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JIAYIN LIU (UNIVERSITY OF JYVÄSKYLÄ): RESTRICTED PROJECTION TO LINES IN $\mathbb{R}^{n+1}$

In $\mathbb{R}^n$, identifying each direction as point in S^{n-1}, Mattila, in 1975, showed that for any Borel set $Z \subset \mathbb{R}^n$, the orthogonal projection of Z to $\mathcal{H}^{n-1}$ almost every direction has dimension $\min\{Z, 1\}$. Recently, there is a series of results studying orthogonal projections to a restricted family of lines in $\mathbb{R}^n$. In this talk, I will summarize the recent progress on restricted projections and report the following restricted projection theorem by myself: Let $M \subset S^n$ be an $(n-1)$-dimensional manifold with sectional curvature > 1. Then for all analytic sets Z, the orthogonal projection of Z to $\mathcal{H}^{n-1}$ almost every direction determined by M has dimension $\min\{Z, 1\}$.

DENIS MARTI (UNIVERSITY OF FRIBOURG): GEOMETRIC AND ANALYTIC STRUCTURES ON METRIC SPACES HOMEOMORPHIC TO A MANIFOLD

We explore geometric and analytic aspects of metric spaces homeomorphic to a closed, oriented manifold. We show that such spaces (which are sometimes called metric manifolds) admit a non-trivial integral current without boundary, provided they satisfy some weak assumptions. The existence of such an object should be thought of as an analytic analog of the fundamental class of the space and can also be interpreted as giving a way to make sense of Stokes' theorem in this setting. We use this to establish (relative) isoperimetric inequalities in metric n-manifolds that are Ahlfors n-regular and linearly locally contractible. As an application, we obtain a short and conceptually simple proof of a deep theorem of Semmes about the validity of Poincaré inequalities in these spaces. We furthermore present applications to the problem of Lipschitz-volume rigidity in the case of metric manifolds. Based on joint work with G. Basso and S. Wenger.

KAISA MATOMÄKI (UNIVERSITY OF TURKU): SIEVE METHODS AND PRIMES IN ARITHMETIC PROGRESSIONS AND SHORT INTERVALS

I will first introduce sieve methods that number theorists use to study primes and almost primes in interesting subsets of natural numbers. Then I will discuss my joint work with Jori Merikoski and Joni Teräväinen where we develop a sieve that can detect primes in multiplicatively structured sets under certain conditions. As applications, I will discuss elementary approaches to primes in short intervals and arithmetic progressions.

MARKUS MYLLYOJA (UNIVERSITY OF OULU): EKSTRÖM-PERSSON CONJECTURE REGARDING RANDOM COVERING SETS

We consider the Hausdorff dimension of random covering sets generated by balls and driven by general measures. We improve a lower bound given by Ekström and Persson in 2018, extend it to generating balls with an arbitrary sequence of radii and prove their conjecture concerning the exact value of dimension in the special case of radii $n^{-\alpha}$ for a certain parameter range. Further, we show that the natural extension of the conjecture is not true for generating balls with an arbitrary sequence of radii. We also give various examples demonstrating the complexity of dimension in this general case. The talk is based on joint work with Esa Järvenpää, Maarit Järvenpää and Örjan Stenflo.

ANNA-MARIYA OTSETOVA (AALTO UNIVERSITY): AXISYMMETRIC CAPILLARY WATER WAVES WITH VORTICITY AND SWIRL CONNECTING TO STATIC UNDULOID CONFIGURATIONS

This is joint work with Erik Wahlén from Lund University, Sweden, and Jörg Weber from University of Vienna, Austria. We study steady axisymmetric water waves with general vorticity and swirl, subject to the influence of surface tension. Explicit solutions to such a water wave problem are static configurations where the surface is an unduloid, that is, a periodic surface of revolution with constant mean curvature. We prove that to any such configuration there connects a global continuum of non-static solutions by means of a global implicit function theorem. To prove this, the key is strict monotonicity of a certain function describing the mean curvature of an unduloid and involving complete elliptic integrals. From this point of view, this paper is an interesting interplay between water waves, geometry, and properties of elliptic integrals.

In the talk, I will present the water wave problem and go over some necessary preliminaries. I will then give a short summary of the proof.

ALEKSI PYÖRÄLÄ (UNIVERSITY OF JYVÄSKYLÄ): SUMMING A SUBSET OF THE PARABOLA WITH ITSELF

Given a set A in the plane, the sumset $A + A = \{a + b : a \in A, b \in A\}$ should in general be substantially larger than the set A, subsequent summations $A + A + ... + A$ even more so. For finite sets, one can expect to discover such a phenomenon between the cardinalities of A and $A + A + ... + A$, and if A is infinite, between the Hausdorff dimensions of A and $A + A + ... + A$. Based on a joint work with Tuomas Orponen and Carmelo Puliatti, I will discuss this phenomenon for subsets of the plane that lie on the parabola $\{(x, x^2) : x \in \mathbb{R}\}$: If A is a subset of the parabola and s denotes the Hausdorff dimension of A, then for any $t < \min\{s + 1, 3s\}$, the Hausdorff dimension of the n-fold sum $nA = A + A + ... + A$ is larger than t for all large enough n. This bound is sharp, and follows from a bound for L^p-norms of Fourier transforms of measures supported on the parabola.

NAGESWARI SHANMUGALINGAM (UNIVERSITY OF CINCINNATI): JUMP SET STRUCTURE OF METRIC SPACE-VALUED BV FUNCTIONS IN METRIC MEASURE SPACES OF 1-CONTROLLED GEOMETRY

Complete metric measure spaces of 1-controlled geometry are spaces where the measure is doubling and supports a 1-Poincaré inequality. In such spaces, real-valued functions of bounded variation, first proposed by Michele Miranda Jr. in his dissertation, have many of the regularity properties that real-valued functions of bounded variation in Euclidean domains enjoy. In 1999 Ambrosio studied metric space-valued functions of bounded variation in Euclidean domains, with primary focus of the target metric space being a subset of a Euclidean space. In this talk we will present analogous results on jump set regularity of metric space-valued mappings of bounded variation on metric measure spaces of 1-controlled geometry.

TIMO TAKALA (AALTO UNIVERSITY): SPHERICALIZATION THAT PRESERVES THE BESOV ENERGY

Sphericalization is the process where we redefine the metric and measure of an unbounded domain such that the domain becomes bounded. It is a generalization of the well known stereographic projection. Transforming the domain in this way is useful for example when studying some PDEs, for in the bounded setting we can use direct methods of the calculus of variations, that don't hold in unbounded domains. In order to apply these methods, we must make sure that all the necessary properties of the space are preserved in sphericalization, such as the doubling property of the measure and the Besov energy of functions. In this talk I present definitions of sphericalization and focus on the specific definition that we ended up studying in order to preserve the Besov energy. This talk is based on joint work with Anders Björn, Jana Björn, Riikka Korte and Sari Rogovin.

ATHANASIOS TSANTARIS (UNIVERSITY OF HELSINKI): QUASICONFORMAL CURVES AND QUASICONFORMAL MAPPINGS

Quasiconformal ω–curves, for some smooth, non-vanishing, closed form ω are embeddings from $\mathbb{R}^n \to \mathbb{R}^m$ satisfying a form of the so called distortion inequality. They can be considered as analogues of classical quasiconformal mappings where the dimension of the domain and the codomain differ. These maps can also be seen as maps from $\mathbb{R}^n \to f(\mathbb{R}^n)$. In this talk we are going to discuss the relationship of these maps with quasiconformal mappings between metric spaces.

NICOLAS VANSPRANGHE (TAMPERE UNIVERSITY): FREQUENCY DOMAIN ESTIMATES FOR ILL-POSED SYSTEMS AND NEUMANN STABILIZATION OF THE WAVE EQUATION

For the multidimensional wave equation $(\partial_t^2 - \Delta)w = 0$ posed in $(0,T) \times \Omega$, it was shown by Irena Lasiecka and Roberto Triggiani that general Neumann boundary data $u \in L^2((0,T) \times \partial\Omega)$ fail to produce finite energy solutions. This is in sharp contrast with the one-dimensional or the Dirichlet input cases, where no such loss of derivative is to be observed.

In the context of linear evolution equations in Hilbert spaces, the lack of well-posedness for the L^2-input problem lessens the effectiveness of control- and system-theoretic methods for stabilization, stability analysis and optimal control. Nevertheless, using the theory of operator semigroups, we are able to establish an exact correspondence between time domain loss of regularity in abstract Sobolev scales and growth rates at high frequencies of certain operator-valued transfer functions.

In turn, the resulting frequency domain estimates provide valuable information for the stability analysis of "damped" contraction semigroups. As an application, we prove new energy decay rates for the wave equation with Neumann boundary feedback under an exact boundary observability condition for the corresponding Schrödinger group.

This is a joint work with Lassi Paunonen (Tampere University) and David Seifert (Newcastle University).

Michele Villa (University of Basque Country): Carleson's ε^2 conjecture in higher dimension and Faber-Krahn inequalities

In this talk I will report on a joint work with Ian Fleschler and Xavier Tolsa on higher dimensional analogues of the Carlesons ε^2 conjecture. In particular, we characterise tangent points of certain domains in Euclidean space via a novel "spherical" square function. Beyond its intrinsic geometric appeal, this result is motivated by connections to quantitative Faber-Krahn inequalities.

Ivan Violo (SNS Pisa): Counting nodal domains of eigenfunctions in non-smooth setting

The Courant nodal domain theorem states that k-th Laplacian eigenfunction with Dirichlet boundary condition has at most k nodal domains. This fact admits an asymptotic version due to Pleijel, which in particular says that this estimate is strict for all but a finite number of eigenfunctions. We extend this result to Neumann Laplacian eigenfunctions on domains with low boundary regularity and allowing the ambient space to be a possibly singular metric space with Ricci curvature bounded below. Based on a joint work with Nicolò De Ponti and Sara Farinelli.

Yu-Liang Wu (University of Oulu): Hausdorff dimension of irreducible Markov chains indexed by d-trees

The talk is based on a joint with Jung-Chao Ban and Guan-Yu Lai, which focuses on the Hausdorff dimensions of irreducible Markov chains indexed by d-trees, a natural generalization of classical Markov chains. In this talk, I will first present a large deviation principle for these generalized Markov chains as an analog of the classical Sanov's theorem, and derive from it a characterization of lower/upper local dimensions of the Markov measures. As an application, I will then apply the results to derive a variational formula for Hausdorff dimensions of the outcome spaces of the aforementioned

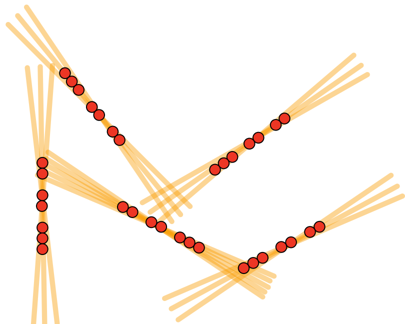
Markov chains. The proof strategy, in spirit, is to transplant the classical thermodynamic formalisms to the Markov subshifts over d-trees.

GUANGZENG YI (UNIVERSITY OF JYVÄSKYLÄ): LARGE CLIQUES IN EXTREMAL INCIDENCE CONFIGURATIONS

Let $P \subset \mathbb{R}^2$ be a Katz-Tao (δ, s)-set, and let $\mathcal{L}$ be a Katz-Tao (δ, t)-set of lines in $\mathbb{R}^2$. A recent result of Yuqiu Fu and Kevin Ren gives a sharp upper bound for the δ-covering number of the set of incidences $\mathcal{I}(P, \mathcal{L}) = \{(p, \ell) \in P \times \mathcal{L} : p \in \ell\}$. In fact, for $s, t \in (0, 2]$,

$$|\mathcal{I}(P, \mathcal{L})|_\delta \lesssim_\epsilon \delta^{-\epsilon - f(s,t)}, \qquad \epsilon > 0,$$

where $f(s, t) > 0$ is an explicit constant. This estimate turns out to be very useful in recent progress for the Furstenberg set problems.



For $s, t \in (0, 1]$, we characterise the near-extremal configurations $P \times \mathcal{L}$ of this inequality: we show that if $|\mathcal{I}(P, \mathcal{L})|_\delta \approx \delta^{-f(s,t)}$, then $P \times \mathcal{L}$ contains "cliques" $P' \times \mathcal{L}'$ satisfying $|\mathcal{I}(P', \mathcal{L}')|_\delta \approx |P'|_\delta |\mathcal{L}'|_\delta$, with the covering number satisfying

$$|P'|_\delta \approx \delta^{-s^2/(s+t)} \quad \text{and} \quad |\mathcal{L}'|_\delta \approx \delta^{-t^2/(s+t)}.$$

Moreover, we show that there exists a list

$$(P_1 \times \mathcal{L}_1), \ldots, (P_n \times \mathcal{L}_n) \subset P \times \mathcal{L}$$

of (δ, δ^u)-cliques, with the sets $\mathcal{D}_\delta(P_j)$ disjoint, and $\sum_j |\mathcal{I}(P_j, \mathcal{L}_j)|_\delta \geqslant \delta^{u - f(s,t)}$. This implies that the only configurations for $P \times \mathcal{L}$ with extremal incidences must contain a sub-configuration as the Figure above.